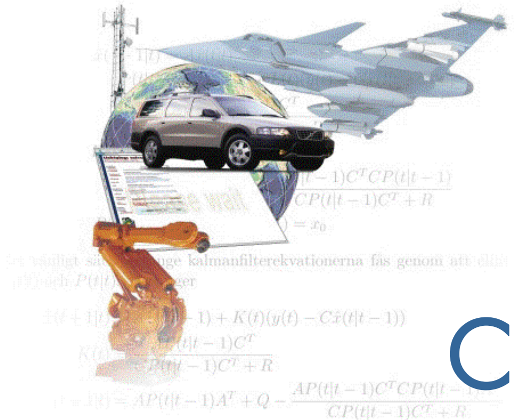


Control of Systems with Input Nonlinearities and Uncertainties: An Adaptive Approach

Ola Härkegård, Torkel Glad
Linköping University

Problem

- Consider a nonlinear system

$$\dot{x} = f(x) + Bg(x,u) \qquad \text{e.g.,} \quad \begin{cases} \dot{x}_1 = f_1(x) \\ \dot{x}_2 = f_2(x) \\ \dot{x}_3 = f_3(x) + g(x,u) \end{cases}$$

- Stabilizing control law:

$$g(x,u) = k(x) \quad \Rightarrow \quad \frac{d}{dt}V(x) = -W(x)$$

Problem

- Uncertain input mapping:

$$g(x,u) = \underbrace{\hat{g}(x,u)}_{\text{known}} + \underbrace{\tilde{g}(x,u)}_{\text{unknown}}$$

?

How can we adjust our control strategy to compensate for $\tilde{g}$?

Aircraft

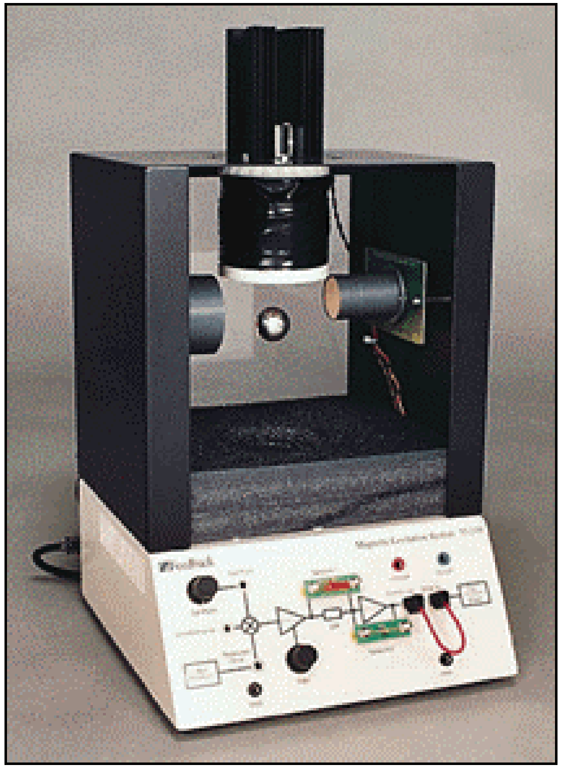
F_T

- Dynamics:

$$\dot{\alpha} = q + \frac{1}{mV}\left(-L(\alpha) + mg\right)$$

$$\dot{q} = \frac{1}{J_y}\left(M(\alpha, q, \delta) + F_T\Delta\right)$$

Levitating ball



- Dynamics:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - \frac{F}{m}$$

- Magnetic force:

$$F = k\frac{(u + U_0)^2}{(x_1 + X_0)^2}$$

Strategy

- Split the model error:

$$\tilde{g}(x,u) = \underbrace{\tilde{g}(r,u_r)}_{\text{cons tan t},\theta} + \underbrace{\tilde{\tilde{g}}(x,u)}_{\text{vanishes}} \approx \theta$$

- Resulting model:

$$\begin{aligned} \dot{x} &= f(x) + B(w + \theta) \\ w &= \hat{g}(x,u) \end{aligned}$$

Control design

$$\dot{x} = f(x) + B(w + \theta)$$
$$w = \hat{g}(x, u)$$

- Ideal control law: $w = k(x) - \theta$
- Certainty equivalence: $w = k(x) - \hat{\theta}$

? How can we estimate θ?

Adaptive backstepping

- Expand the Lyapunov function.

$$V_a\left(x,\tilde{\theta}\right)=V(x)+\frac{\gamma}{2}\tilde{\theta}^2,\qquad \tilde{\theta}=\theta-\hat{\theta}$$

- Demand $\dot{V}_a=-W(x)+\left(V_xB-\gamma^{-1}\dot{\hat{\theta}}\right)\tilde{\theta}=-W(x)$

- Choose $\dot{\hat{\theta}}=\gamma V_xB$

- Control law: $\quad w=k(x)+\gamma\int V_xBdt$

Nonlinear observer

- Extend the system dynamics:

$$\begin{cases} \dot{x} = f(x) + B(w + \theta) \\ \dot{\theta} = 0 \end{cases}$$

- Observer:

$$\begin{cases} \dot{\hat{x}} = f(x) + B\left(w + \hat{\theta}\right) + K_1(x - \hat{x}) \\ \dot{\hat{\theta}} = 0 + K_2(x - \hat{x}) \end{cases}$$

- Control law: $w = k(x) - \hat{\theta} - \gamma V_x B$

Levitating ball

- Dynamics: $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = w + \theta \end{cases}$$

  where $$w = g - \frac{k}{m} \cdot \frac{(u + U_0)^2}{(x_1 + X_0)^2}$$

- Control law: $$w = -l_1(x_1 - r) - l_2 x_2 - \hat{\theta}$$

Estimator alternatives

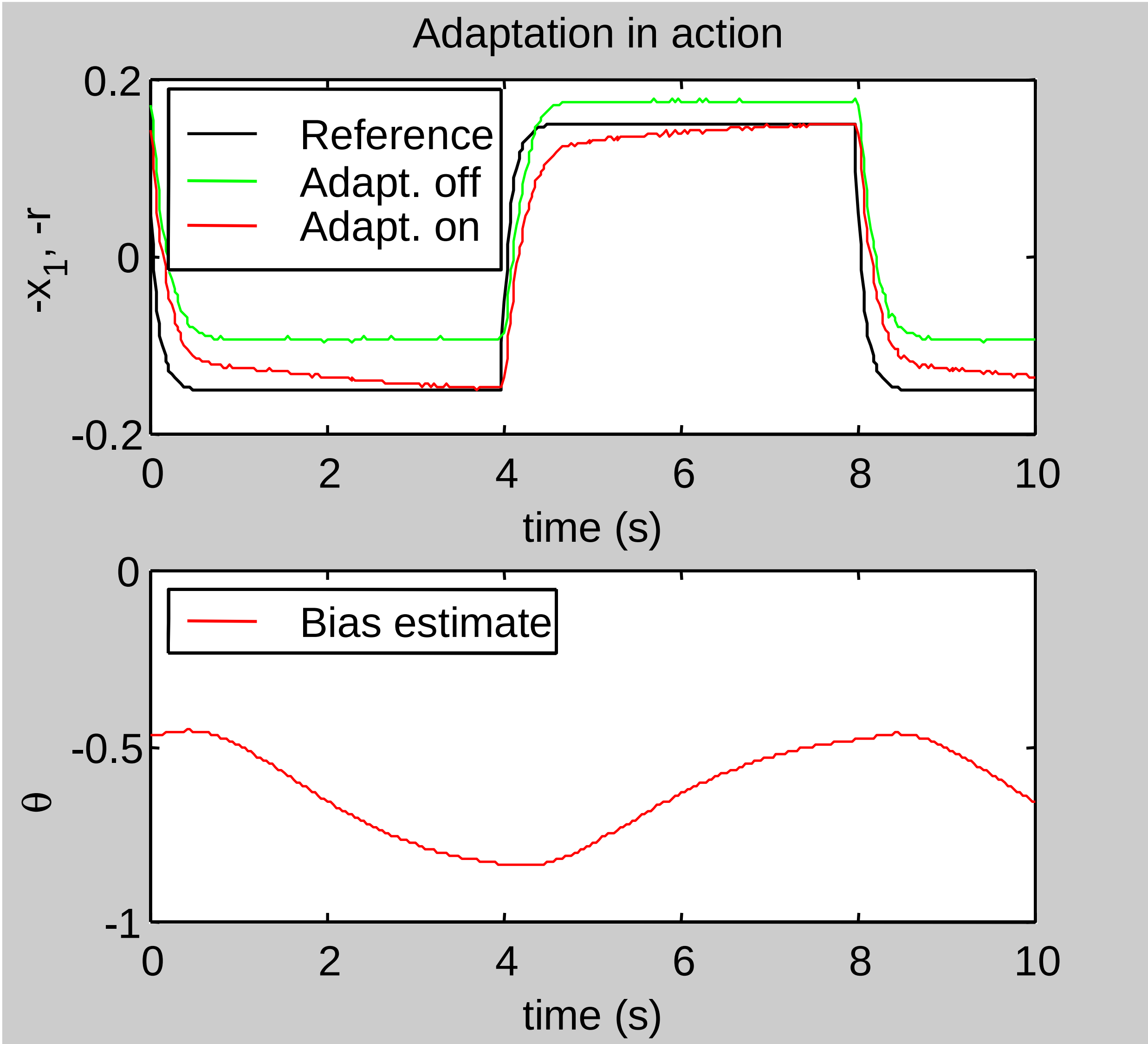
- Adaptive backstepping

$$\hat{\theta} = \int \gamma_1 (x_1 - r) + \gamma_1 x_2 dt$$

- Nonlinear observer

$$\hat{\theta} = \frac{k_2}{s^2 + k_1 s + k_2} \cdot (s x_2 - w)$$

Experimental results



Attaching a second ball