Backstepping Control of a Rigid Body

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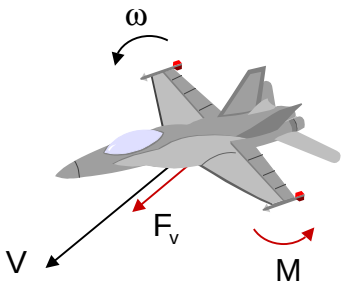
Control Problem

- Outputs: V, ω
- Inputs: F_v, M



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AUTOMATIC CONTROL
COMMUNICATION SYSTEMS
LINKÖPINGS UNIVERSITET

Dynamics

- Rigid body dynamics

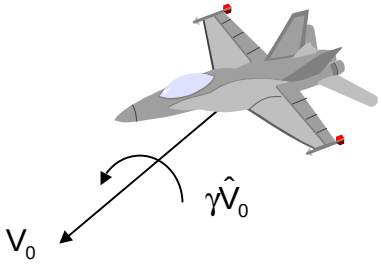
$$\begin{aligned}\dot{V} &= -\omega \times V + \frac{1}{m}F \\ J\dot{\omega} &= -\omega \times J\omega + M\end{aligned}$$

where $F = m\left(F_a(V) + u_v \hat{V}\right)$

Stationary Motion

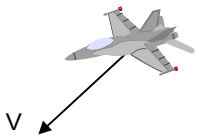
- $V = V_0,\ \omega = \omega_0$ gives

$$\omega_0 = \frac{1}{|V_0|}\left(\hat{V}_0 \times F_a(V_0)\right) + \gamma \hat{V}_0$$



Backstepping Control Design (step 1)

- Dynamics: $\dot{V} = -\omega \times V + F_a + u_v \hat{V}$



- Control variables: u_v, ω
- Lyapunov function: $W_1 = |V - V_0|^2$

$$u_v = k_1 (V_0 - V)^T \hat{V} - V^T F_a$$
$$\omega^{des} = k_2 (V_0 \times V) + \frac{1}{|V|^2} V \times F_a + \gamma \hat{V}_0$$

Backstepping Control Design (step 2)

- $\xi = \omega - \omega^{des} \Rightarrow$

$$\dot{V} = \dots$$
$$\dot{\xi} = u_M - \phi$$

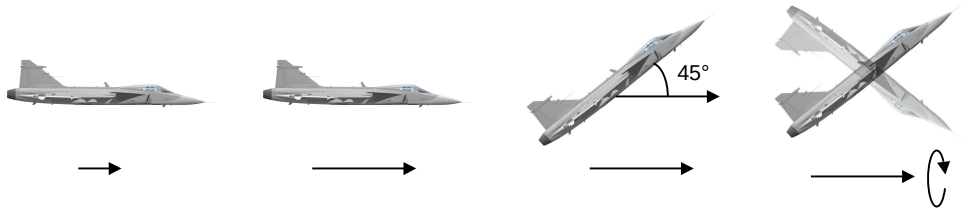
$$u_M = \dot{\omega} = J^{-1}(M - \omega \times J\omega)$$
$$\phi = \dot{\omega}^{des}$$

- Control variable: u_M
- Lyapunov function: $W_2 = k_3 W_1 + |\xi|^2$

Control law:

$$u_M = k_3 V_0 \times V - k_4 \xi + \phi$$
$$u_v = k_1 (V_0 - V)^T \hat{V} - V^T F_a$$

Simulated Maneuver



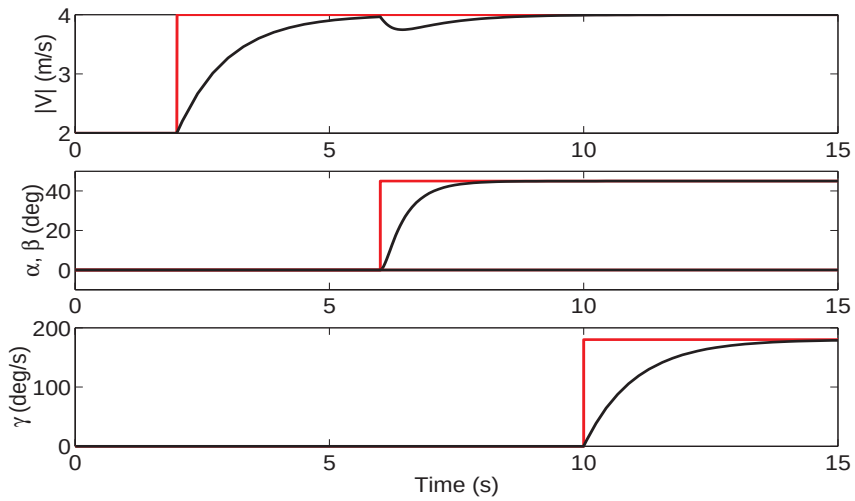
$$V_0 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \quad \gamma = 0$$

$$V_0 = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \quad \gamma = 0$$

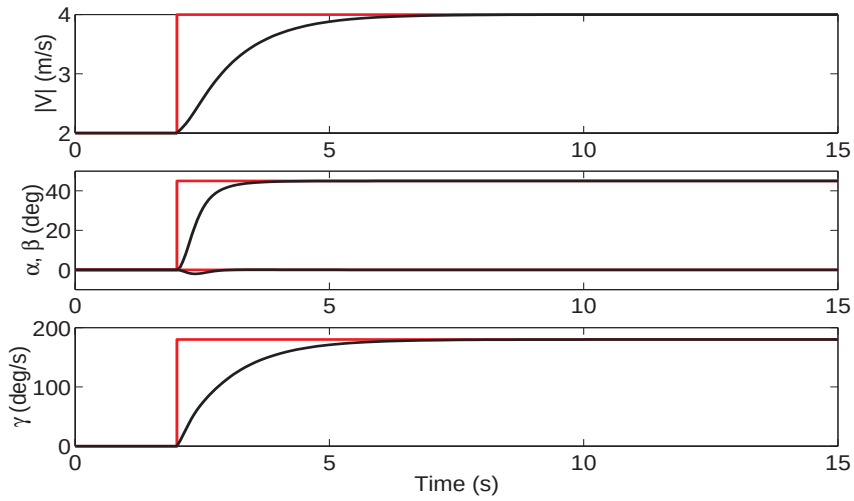
$$V_0 = \sqrt{2}\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \quad \gamma = 0$$

$$V_0 = \sqrt{2}\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \quad \gamma = \pi$$

Simulation Results



Simulation Results



Conclusions

- Rigid body control using
  - backstepping
  - vector form
- Global stability
- Good decoupling